Conservative Derivatives

# The conservative 1D Laplacian

MOONS computes

on a staggered grid for both cell corner and cell centered data. Let subscript and represent the primary and dual grid of the staggered grid respectively. If the data lives on the primary grid, then the result also lives on the primary grid, and the coefficient, , lives on the dual grid. This derivative is computed as follows

This form of the derivative is explicitly clear and consistent (since all references are to the primary grid). First, let's be clear about the current index situation in MOONS:

First note that lives on integers of for BOTH CC and N based data.

## Cell center (C) data

u1 u2 ... ui usc

o---**|**---**o**---**l**---o---l---o---l ---o---l ---o---l ---o---|---o

Let of the primary grid be of the dual grid (this can easily be seen by replacing with 1). NOTE the ghost cell.

This means that if then refers to the first index of which is (what we want). This verifies the index for the CC data case.

## Cell corner, or node (N), data

u1 u2 ... ui ... usn

o---|---**o**---**l**---**o**---l---o---l ---o---l ---o---l ---o---|---o

Let of the primary grid be of the dual grid (this can easily be seen by replacing with 1).

This means that if then refers to the *second* index of which is (what we want). This verifies the index for the N data case.

# General form without half indexes

Now, we can write a more general equation

Where

Now we have successfully removed the half indexes, and this form is easily programmable. Note that has also adopted this index convention as well. Furthermore note that, from the second equation, it is clear that . Now, since

let for both grids, and we have

# Computing k

The coefficient must be linearly interpolated when , otherwise an ordinary average suffices. The formulation of may be determined from

Therefore we may compute to be

Note that if , then we have and and so

This is a general form of the conservative 1D Laplacian stencil for BOTH cell centered and cell corner data with all half indexes removed.